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RANDOM PARAMETRIC EFFECTS IN CERTAIN PROBLEMS OF AEROELASTICITY

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RANDOM PARAMETRIC EFFECTS IN CERTAIN PROBLEMS OF AEROELASTICITY

(Parametricheskiye sluchaynyye vozdeystviya v nekotorykh zadachakh aerouprugosti)

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When determining the dynamic reactions to atmosphere turbulence in near-critical states (divergence, flutter), the accounting of the fluctuations of the flight's near-average horizontal speed relatively to the air is of interest. These horizontal speed fluctuations are conditioned by the atmosphere turbulence, and they have the character of a random vertical component of the turbulence velocity. It follows from [1], that the respective diagrams of spectral density curves and of the vertical component introduced in [1, 2], may be used when when calculating the spectral density of the horizontal component of turbulence velocity.

In the following, we shall describe a method of evaluation of random parameters' effect on the behavior of the vibrating system (flexible aircraft - air). It is shown that the near-critical states (divergence, flutter) and, on account of atmosphere turbulence, the fluctuations of aircraft's horizontal speed, reduce the reserve of system's stability and may induce a parametric disturbance. The evaluation of the effect of variable parameters on a system with many degrees of freedom does not pretend to be of high precision, and it demands a preliminary investigation of the system's natural frequency behavior in relation to parameters.

1. Problem of Torsional Vibrations of a High-Aspect-Ratio Aerofoil in Turbulent Flight Conditions

In order to clarify the role of variable parameters in near critical

states, let us examine first of all the particular problem of torsional vibrations of a high aspect ratio aerofoil in turbulent flight conditions. Let us then consider that the horizontal component of the speed of flight is subsonic, that the wing's rigidity on bend is infinite, and that the mass of the fuselage is also infinitely large compared to the mass of the wing. Let us limit ourselves further to considering the f rst mode of symmetric torsional vibrations of the wing. Then the wing may be substituted by an equivalent aerodynamic profile in a steady flow, "hinge-secured" at a point O_1, which corresponds to a fixed axis about which the elastic wing twists, as if it were a spring, with a rigidity represented by $\varkappa$. The scheme of the profile is presented in figure 1, where b is the length of the chord, σ' is the distance between O and O_1; $U + \xi(t)$ is the horizontal flow velocity; U is the mean value of that flow. Let $\xi(t)$ be the random component with a mean value $\bar{\xi}(t) = 0$. We may note that in real cases, the root-mean- square value $(\overline{\xi^2})^{1/2} \ll U$.

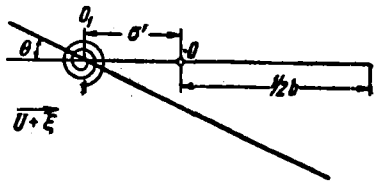

Fig. 1

The equation of profile's torsional vibrations may be written in the form

$$\ddot{\theta} + \dot{\theta}\left[\pi\rho\,(U + \xi(t))\,\Delta_1\left(\frac{1}{4} - \Delta_2\right)\frac{S_1}{J}\right] + \theta\left[\omega_\theta^2 \pi\rho\,(U + \xi(t))^2\,\Delta_2\frac{S_2}{J}\right] = M(t) \qquad (1)$$

Here θ is the angle of the twist,

$$\Delta_1 = \frac{1}{2}\left(\frac{1}{2} - a\right), \quad \Delta_2 = \frac{1}{2}\left(\frac{1}{2} + a\right), \quad a = \frac{2\sigma'}{b}, \quad \omega_\theta^2 = \frac{\varkappa}{J}$$

J is the profile's moment of inertia, S_1 and S_2 are coefficients determinable by the wing's geometry and by the form of torsional vibrations; ρ is the density of the air; $JM(t)$ is the moment of random aerodynamic forces occurring through the effect of turbulence velocity vertical component. Let us note that the equation (1) is written with the assumption that $C(k) = 1$, where $C(k)$ is the Theodorsen function [2].

Let us introduce the following designations:

$$\delta = \frac{U}{2}\pi\rho\Delta_1\left(\frac{1}{4} - \Delta_2\right)\frac{S_1}{J}, \quad \Omega = \sqrt{\omega_\theta^2 - \pi\rho\frac{U^2\Delta_2 S_2}{J}}, \quad \varepsilon = \frac{2U\Delta_2 S_2}{J\Omega^2} \qquad (2)$$

Then, equation (1) will be written with a precision to magnitudes of the order of $(\overline{\xi^2})^{1/2}/U$ in the form

$$\ddot{\theta} + 2\delta\left(1 + \frac{\xi(t)}{U}\right)\dot{\theta} + \Omega^2[1 + \varepsilon\xi(t)]\theta = M(t) \qquad (3)$$

The equation (1) – (3) actually is the linear equation with fluctuating coefficients.

It is shown in [3] that the effect of coefficient's fluctuations determining the system's natural frequency, is equivalent to the introduction in the examined vibrating system of negative damping

$$\delta_* = \frac{\varepsilon^2\Omega^2 S_\xi(2\Omega)}{16} = \frac{4U^2\Delta_2 S_2 \pi\rho}{J(U_{**}^2 - U^2)} S_\xi(2\Omega) \qquad \left(U_{**} = \sqrt{\frac{J}{\pi\rho\Delta_2 S_2}}\right) \qquad (4)$$

Here U_{**} is the critical divergence speed, and $S_\xi(2\Omega)$ is the spectral density of the $\xi(t)$ process at a frequency 2Ω. We neglect the influence of the function $\xi(t)/U$ in the second term of equation (3) for in all practically realizable cases the magnitude

$$S^2/U^2 \ll \Omega^2\varepsilon^2$$

At the same time, the spectral density $S_\theta(\omega)$ of the function $\theta(t)$ may be approximately represented by means of spectral density $S_M(\omega)$ of the function M(t) with the help of the expression

$$S_\theta(\omega) = \frac{S_M(\omega)}{4(\delta - \delta_*)^2 + (\omega^2 - \Omega^2)^2} \qquad (5)$$

Formula (5) is valid in the case when the observation of the system is conducted during a longer period than the relaxationttime of the system $1/\beta$, and under the condition that the random functions M(t) and $\xi(t)$ are not mutually correlated (this is fulfilled, for the vertical and horizontal turbulence velocities are practically independent). The function $S_M(\omega)$ is easy to determine from the turbulence spectrum with the aid of function introduced in reference [2].

If the relation $\delta_* \geqslant \delta$ is fulfilled, the torsional vibrations become parametrically perturbed, and they may be limited only by the nonlinearity of the real system [3].

TABLE 1.

$\frac{U}{U_{**}}$	a	$J = 50$ кг·м²			$J = 100$ кг·м²			$J = 200$ кг·м²		
		U_{**} м/сек	δ	δ_*	U_{**} м/сек	δ	δ_*	U_{**} м/сек	δ	δ_*
0.95	—0.04	68	0.35	0.03	96	0.24	0.06	136	0.17	0,17
	—0.1	73	1.03	0.02	103	0.73	0.06	145	0.51	0.14
	—0.2	84	2.73	0.02	119	1.96	0.06	168	1.38	0.10
0.98	—0.04	68	0.36	0.17	96	0.25	0.32	136	0.18	0.80
	—0.1	73	1.06	0.14	103	0.75	0.34	145	0.53	0.80
	—0.2	84	2.86	0.10	119	2.03	0.26	168	1.48	0.92
0.95	—0.04	40	0.21	0.05	57	0.15	0.17	81	0.10	0.28
	—0.1	44	0.62	0.05	61	0.43	0.11	87	0.31	0.24
	—0.2	50	1.65	0.04	71	1.17	0.08	100	0.83	0.18
0.98	—0.04	40	0.21	0.34	57	0.16	1.50	81	0.11	1.98
	—0.1	44	0.64	0.40	61	0.45	0.50	87	0.32	1.50
	—0.2	50	1.70	0.19	71	1.20	0.40	100	0.85	0.77

(In the upper part of Table 1 the values of parameters are given for , and in the lower part, they are given for $\omega_\vartheta = 2\pi\ 3.00$ rad/sec)

Table 1 presents the magnitudes δ and δ_* for different values of parameters U, U_{**} (m/sec), ω_0, J kg/cm^2 and a. The magnitudes S_1 and S_2 respectively are : $S_1 = 24.5\ m^4$, $S_2 = 11.6\ m^3$, just as presented in reference [2]. It follows from the Table that the magnitude δ is decreasing with the increase of the relation U/U_{**}, while a is diminishing, with the increase of the magnitude J, and with the decrease of the natural frequency of torsional vibrations. The magnitude δ_* behaves contrarywise. That is why the variation of parameters U/U_{**}, a, J and ω_ϑ, in the indicated direction leads to a decrease in the reserve of system's stability, and it may even induce a parametric perturbation in the vibration of the wing.

The data provided by the Table correspond to the turbulence level with root-mean-square values of the magnitude of the wind gust velocity exceeding seven times the corresponding magnitudes presented in [1].

2. Problem of Airfoil Vibration in a Turbulent Flow for numerous Degrees of Freedom

When studying flutter phenomena, one may sometimes have recourse to the representation of trajectories of the vibrational system's complex frequencies (flexible aircraft - air) on a complex plane with axes δ and $j\omega$. Let us make use of a similar representation to estimate the influence of variable parameters upon the complex vibration system.

Let us assume that the pattern of magnitudes δ and ω variations relatively to certain parameters, such as U, is known. Such pattern is plotted in figure 2, which is borrowed from reference [4]. It is obtained for a model of aircaraft accounting for two vibrational degrees of freedom. K designates the value of the dimensionless velocity. If the velocity is somewhat deflected from its mean value U, it is natural to express the magnitudes δ_i and ω_i, characterizing the vibration of the ith coordinate of the examined system in the form

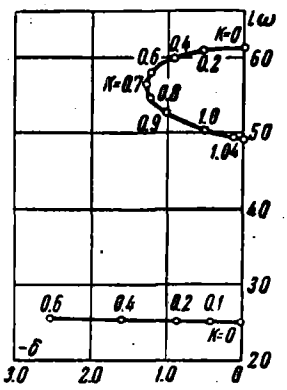

Fig. 2

$$\omega_i = \omega_i(\xi), \qquad \delta_i = \delta_i(\xi) \tag{6}$$

(t) being the small velocity deflections.

The influence of small paramater variations on the parametric stability of the examined vibrationa; system may be practically expressed for those sectors of the complex natural frequency trajectories, which lay in the narrow band along the imaginary axis of the complex plane. At the same time, if the natural frequencies ω_i and ω_k for equal values of the parameter U on the complex plane are situated sufficiently far from one another, say

$$|\omega_i - \omega_k| \gg \delta_i, \qquad |\omega_i - \omega_k| \gg \delta_k$$

or if δ_i and δ_k strongly differ from one another, the problem of parametric stability may be separately raised for each coordinate.

Taking into account the expressed assumptions, the variation of each i-th coordinate may be approximately described with the help of vibration system equations of the second order, whose coefficients are

determined by the magnitudes ω_i and δ_i. This equation will have the form

$$\ddot{y}_i + 2\delta_i [U, \xi(t)]\dot{y}_i + \omega_i^2 [U, \xi(t)] y_i = 0 \qquad (7)$$

If $\xi(t)$ is the periodical function of time (sinusoidal for example), we are faced with the problem of parametric resonance in a linear vibration system. If $\xi(t)$ is a random function of time, we are in front of a problem already examined in the first part of the present paper. The first case may have a practical interest during the calculation of the dynamic resistance of a helicopter blade, since the horizontal velocity of air flow relatively to the blade is undergoing a considerable sinusoidal variation with the frequency of propeller's rotation.

The second case offers interest in the determination of the influence of atmosphere turbulence velocity's horizontal component on the near-flutter vibrations of the wing of an aircraft. Here, the approximate estimate of the magnitude of negative damping, introduced as a result of velocity fluctuation may be made. For $\xi(t)$ values, much smaller ... U, ω_i^2 $[U, \xi]$ may be written in the form

$$\omega_i^2 [U, \xi(t)] = \omega_{i0}^2 (v) [1 + \varepsilon_i \xi(t)] \qquad (8)$$

where $\omega_{i0}^2 (U)$, ε_i do not depend on time.

The greatest influence of the variable parameters on the behavior of the vibrational system must be expected in the presence of natural frequencies, whose trajectories approach the imaginary axis at a sharp angle (say at greater ε_i values). For a system with trajectories shown in fig. 2, the case is exactly opposite. For both, the first and the second natural frequency near the imaginary axis $\varepsilon_i = 0$, and that is why the fluctuations do not exert any influence on the stability of the system.

The δ_i variation at speed fluctuations will be reflected ... on the system's parametric stability only at problem examination when account is taken of the magnitude of the second order of smallness $\varepsilon^2 \overline{\xi^2} / U^2$ and that is why they may be neglected.

In conclusion, let us make one remark concerning the character of atmosphere turbulence. It follows from [5] and other sources, that the turbulence in atmosphere is not the same everywhere. During its flight, an aircraft meets separate areas with considerable levels of turbulence. That is why its influence in those areas may result in more considerable effects, as it follows from the average data covered in papers [1 and 2]. For a still more precise evaluation of this effect, its unsteady character ought to be taken into account.

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*** END ***

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References follow...

REFERENCES.

1. PRESS H., HOUBOLT J. C. Somme Applications of Generalized Harmonic Analysis to Gust Load on Airplanes. J. Aeronaut. Sci., v. 22. No. 1, pp. 17-26, 1955.

2. ROMANOVSKIY Yu. M., STRELKOV C. P. O. O vozdeystvii atmosfernoy turbulentnosti na samolet s uprugimi kryl'yami pri razlichnykh skorostyakh poleta. (On the Effect of Atmosphere Turbulence upon an Aircraft with Felxible Wings and for Different Flight Speeds). Izv. A. N. SSSR, OTN, Mekhanika i Mashinostr., 4, 1959.

3. STRATONOVIC R. L., ROMANOVSKIY Yu. M. Parametricheskoye vozdeystviye sluchaynoy sily na lineynyye i nelineynye kolebatel'nyye sistemy, (Parametric Effect of a Random Force on Linear and Nonlinear Vibrational Systems). NDVSh, ser. phys., math., sci, NO. 3, pp. 221-224, 1958.

4. KHARLAMOV A. A. Modelirovaniya aerodinamicheskikh vozdeystviy na kolebayushcheyesya krylo v ploskom potoke vozdukha. (Aerodynamic Effect Modeling on Vibrating Wing in a Plane Air Flow). - Do. pp. 149-154.

5. KURINOVA Yu. V., KHACHATARYAN S. P. O strukture atmosfernoy turbulentnosti po dannym skorostnykh samoletov. (On the Structure of Atmospheric Turbulence from Data of Speeding Aircrafts). Meteorologiya i Gidrologiya, No. 8, 1959.